

**Chapter 5**

**Squaring and square Roots**

# Square of numbers ending in 5 :

**Sutra:** ‘By one more than previous one”

**Example:** 75 × 75 or 752

As explained earlier in the chapter of multiplication we simply multiply 7 by the next number i.e. 8 to get 56 which forms first part of answer and the last part is simply 25= (5)2. So, 75 × 75 = 5625

This method is applicable to numbers of any size.

**Example**: 6052

60 × 61 = 3660 and 52 = 25

 6052 = 366025

Square of numbers with decimals ending in 5

**Example** : (7.5)2

7 × 8 = 56, (0.52) = 0.25

(7.5)2 = 56.25 (Similar to above example but with decimal) Squaring numbers above 50:

**Example:** 522

**Step1:** First part is calculated as 52 + 2 = 25 + 2 = 27

**Step2:** Last part is calculated as (2) 2 = 04 (two digits)

 522 = 2704

# Squaring numbers below 50

**Example** : 482

**Step1:** First part of answer calculated as: 52 – 2 = 25 – 2 = 23

**Step2:** second part is calculated as : 22 = 04

 482 =2304

# Squaring numbers near base :

**Example :** 10042

**Step1:** For first part add 1004and 04 to get 1008

**Step2**: For second part42 = 16 = 016 (as,base is 1000 a three digit no.)

 (1004)2 = 1008016

**1**

# Squaring numbers near sub - base:

**Example** (302)**2**

**Step1:** For first part = 3 (302 + 02) = 3 × 304 = 912 [Here sub – base is 300 so multiply by 3]

**Step2:** For second part = 22 = 04

 (302)2 = 91204

# General method of squaring:

**The Duplex**

**Sutra:** “Single digit square, pair multiply and double” we will use the term duplex,` D’ as follows: For 1 figure(or digit) Duplex is its squaree.g. D(4) = 42 = 16

For2 digitsDuplex is twice of the product e.g. D(34) = 2 (3 x 4) = 24 For 3 digit number: e.g. (341)2

D(3) = 32 = 9

D (34) = 2 (3 × 4) = 24

D (341) = 2 (3 × 1 ) + 42 = 6 + 16 = 22

D (41 ) = 2 (4 × 1 ) = 8

9 4 2 8 1

2 2

D (1) = 12 = 1 =116281

 (341)2 = 116281

# Algebraic Squaring :

Above method is applicable for squaring algebraic expressions:

**Example:** (*x* + 5)2 D (*x*) = *x*2

D(*x* + 5) = 2 (*x* × 5) = 10*x* D (5) = 52 = 25

 (*x* + 5)2 = *x*2 + 10*x* + 25

**Example:** (*x* – 3*y*)2 D (*x*)= *x*2

D(*x* – 3*y*) = 2 (*x*) × – 3*y*) = – 6*xy*

D(–3*y* ) = (–3*y*)2 = 9*y*2

 (*x* – 3*y*)2 = *x*2 – 6*xy* + 9*y*2

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Try these:** |  | | | | | |
| (I) 852 | (II) | 1 2  (82 ) | (III) | (10.5)2 | (IV) | 80502 |
| (V) 582 | (VI) | 522 | (VII) | 422 | (VIII) | 462 |
| (IX) 982 | (X) | 1062 | (XI) | 1182 | (XII) | (*x* + 2 )2 |
| (XIII) (y – 3)2 | (XIV) | (2*x* – 3)2 | (XV) | (3*y* – 5)2 |  |  |

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# SQUARE ROOTS:

**General method:**

As 12 = 1 22 = 4 32 = 9 42 = 1[ 6 ] 52 = 2 [5] 62 = 3 [6]

72 = 4 [9] 82 = 6 [4] 92 = 8[1] i.e. square numbers only have digits 1,4,5,6,9,0 at the units place (or at the end)

Also in 16, digit sum = 1 + 6 = 7, 25 = 2 + 5 =7 , 36 = 3 + 6 = 9 , 49 = 4 + 9 = 13

13 = 1 + 3 =4, 64 = 6 + 4 = 10 = 1 + 0 = 1 , 81 = 8 + 1 = 9 i.e. square number only have digit sums

of 1, 4, 7 and 9.

This means that square numbers cannot have certain digit sums and they cannot end with certain figures (or digits) using above information which of the following are not square numbers:

(1) 4539 (2) 6889 (3) 104976 (4) 27478 (5) 12345

**Note:** If a number has a valid digit sum and a valid last figure that does not mean that it is a square number. If 75379 is not a perfect square in spite of the fact that its digit sum is 4 and last figure is 9.

# Square Root of Perfect Squares:

**Example1**: 5184

**Step 1:** Pair the numbers from right to left 5184 two pairs Therefore answer is 2 digit numbers

72 = 49 and 82 = 64

49 is less than 51

Therefore first digit of square root is 7. Look at last digit which is 4

As 22 = 4 and 82 = 64 both end with 4 Therefore the answer could be 72 or 78 As we know 752 = 5625 greater than 5184 Therefore 5184 is below 75

Therefore 5184 =72

**Example 2:** 9216

**Step 1:** Pair the numbers from right to left 9216two pairs Therefore answer is 2 digit numbers

92 = 81 and 102 = 100

81 is less than 92

Therefore first digit of square root is 9. Look at last digit which is 6

As 42 = 16 and 62 = 36 both end with 6 Therefore the answer could be 94 or 96

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As we know 952 = 9025 less than 9216 Therefore 9216 is above 95

Therefore 9216 = 96

# General method

**Example 1** : 2809

**Step1:** Form the pairs from right to left which decide the number of digits in the square root. Here 2 pairs therefore 2 - digits in thesquare root

**Step 2:** Now 28, nearest squares is = 25 So first digit is 5 (from left)

**Step3:** As 28 – 25 = 3 is reminder which forms 30 with the next digit 0.

**Step 4:** Multiply 2 with 5 to get 10 which is divisor 10 2809

30

Now 3 × 10 = 30 30 = Q R

10 3 0

**Step 5:** As 32 = 9 and 9 – 9 (last digit of the number) = 0

 2809 is a perfect square and 2809 = 53

**Example 2:**3249

**Step1:** Form the pairs form right to left which decided the number of digits in the square root. Here 2 pairs therefore 2 digits in the square root.

**Step2:** Now 32 > 25 = 52 so the first digit in 5 (from left)

**Step 3:** 32 – 25 = 7 is remainder which form 74 with the next digit 4

5 7

**Step 4:** Multiply 2 with 5 to get 10 which is divisor 103249

Now 74 = Q R 7 4

107 4

**Step5:** 72 = 49 and 49 – 49 =0 (remainder is 4 which together with9 form49)

 3249 is a perfect square and 3249 = 57

# Example 3:

54 75 6

**Step1**: Form the pairs from right to left therefore the square root of 54756 has 3-digits.

**Step2:** 5 > 4 = 22 i.e. nearest square is 22 = 4 So first digit is 2 (from left)

**Step3:** As 5 – 4 = 1 is remainder which form 14 with the next digit 4.

**4**

**Step4:** Multiply 2 with 2 to get 4, which is divisor 2

4 514 275 6 Now 14 = Q R

4 3 2

**Step 5:** Start with remainder and next digit, we get 27. Find 27 – 32 = 27 – 9 = 18 [square of quotient]

234

**Step 6**: 18 = Q R 4 5142 7251 6

4 4 2

Now 25 – (3 × 4 × 2) = 25 – 24 = 1

1 = Q R

4 0 1

16 – 42 = 16 – 16 = 0

 54756 is a perfect square and so 5 4 7 5 6 = 234

# Try These:

|  |  |  |  |
| --- | --- | --- | --- |
| 1. | 2116 | 2. | 784 |
| 3. | 6724 | 4. | 4489 |
| 5. | 9604 | 6. | 3249 |
| 7. | 34856 | 8. | 1444 |
| 9. | 103041 | 10. | 97344 |

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